

The Role of Strange and Charm Quarks in the Nucleon Spin Structure Function

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Abstract

We perform an analysis of the relation between the factorization scale and the masses of the quarks in the calculation of the hard gluon coefficient in polarized deep inelastic scattering. Particular attention is paid to the role of strange and charm quarks at finite momentum transfer. It is found that for the momentum transfer of the present experiments, the contribution from the charm quark is significant.

1 Introduction

In the usual analysis of the proton spin structure function, based upon QCD and the operator product expansion (OPE), the moments of the singlet part of $g_1(x, Q^2)$ are written as:

$$\int_0^1 g_1^{(s)}(x, Q^2) x^{n-1} dx = \Delta\Sigma_n(\mu^2) C_n^q(Q^2/\mu^2) + \Delta g_n(\mu^2) C_n^g(Q^2/\mu^2), \quad (1)$$

with $C_n^q(Q^2/\mu^2) = 1 + O(\alpha_s)$ the Wilson coefficients for the quark operators, $C_n^g(Q^2/\mu^2) = O(\alpha_s)$ the Wilson coefficients for the gluon operators. The matrix elements $\Delta\Sigma_n(\mu^2)$ and $\Delta g_n(\mu^2)$ are not determined by perturbative QCD and should be either fixed by experimental constraints or calculated

using non-perturbative techniques. Eq. (1) can be inverted, using the inverse Mellin transformation, and the result is:

$$g_1^{(s)}(x, Q^2) = \Delta\Sigma(x, \mu^2) \otimes C^q(x, Q^2/\mu^2) + \Delta g(x, \mu^2) \otimes C^g(x, Q^2/\mu^2), \quad (2)$$

where \otimes denotes a convolution of the two functions.

Much of the debate on the proton spin in the last few years has been centered on whether or not the spin of the proton receives a contribution from the gluons [1-6]. On the basis of the OPE the picture is clear: there is no twist two gluon operator contributing to the first moment of g_1 , and hence $\int_0^1 C^g(x, Q^2/\mu^2) dx = 0$. This result implies that the first moment of g_1 is given solely by the first moment of $\Delta\Sigma$. If $\Delta\Sigma$ were identified with the spin in the proton carried by the quarks then the gluons would give no contribution. In this scenario, following the parton model language, $\Delta\Sigma_n(\mu^2) = N \sum_f \Delta f_n(\mu^2)$, with $\Delta f = \Delta f_1$ the amount of spin carried by the f quark and N equals 1/9 for three flavors, 5/36 for four flavors, etc. It happens that $\Delta\Sigma$ cannot be identified with spin because of the axial anomaly. Indeed, the axial anomaly is at the heart of the disagreement between the OPE and the improved parton model (IPM) results for the role of gluons in the first moment of $g_1(x, Q^2)$. In this contribution, we are not going to make a complete analysis of the equivalence (or otherwise) of these approaches but will limit ourselves to the analysis of the gluon contribution in the light of the IPM only.

In the IPM the situation is more complicated. One calculates the full, polarized photon-proton cross section and uses the factorization theorem to separate the hard and soft parts:

$$\sigma^{\gamma^* N}(x, Q^2) = \sigma_h^{\gamma^* q}(x, Q^2/\mu^2) \otimes \Delta f_{q/N}(x, \mu^2) + \sigma_h^{\gamma^* g}(x, Q^2/\mu^2) \otimes \Delta f_{g/N}(x, \mu^2), \quad (3)$$

where μ^2 is the factorization scale, $\Delta f_{q(g)/N}$ is the polarised quark (gluon) spin distribution inside the nucleon and σ_h is the polarized, hard photon-quark or hard photon-gluon cross section. One then could relate g_1 calculated in the IPM, Eq. (3), to g_1 calculated in the OPE, Eq. (2), by identifying the hard, perturbatively calculated, Wilson coefficients with the hard photon-quark and hard photon-gluon cross sections and identifying the matrix element $\Delta\Sigma(x, \mu^2)$ ($\Delta g(x, \mu^2)$) with the factorized quark (gluon) distribution

$\Delta f_{q(g)/N}$. However, as already mentioned, $\Delta\Sigma(x, \mu^2)$ cannot be identified with the quark spin distribution. The relation between them is beyond the scope of the present work. Instead, we will concentrate on the relation between the Wilson gluon coefficient and the hard gluon cross section of the IPM. Although there are excellent treatments of this subject in the literature [2, 6, 7, 8], we think that the present contribution adds significantly to the understanding of the behaviour of $g_1(x, Q^2)$ at finite Q^2 .

In section 2 we will develop the basis for the calculation of the hard gluon coefficient in the IPM. The resulting expression interpolates the known limits of $-\frac{\alpha_s}{2\pi}N_f$ for $m_q^2 \ll \mu^2$ and 0 for $m_q^2 \gg \mu^2$ and overcomes convergence problems found in an early work [22]. The effects of this generalized, hard gluon coefficient are discussed in section 3. In particular, its effect on the contribution to $g_1^p(x, Q^2)$ from up, down, strange and charm quarks is studied. Our results indicate that these corrections are sizable and must therefore be taken into account when extracting the polarized gluon distribution from the proton. We also point out in this section how this anomalous contribution is affected by finite Q^2 . In section 4 we calculate the amount of polarized gluon in the proton necessary to explain the available data. We compare our result with other estimates made using simply the limiting cases for the hard gluon coefficients. Section 5 is used to study the region in x where this contribution is located. In section 6 we summarise the results obtained in this article.

2 Theoretical Construction

The hard gluon cross section is extracted from the full photon-gluon fusion cross section, σ^{γ^*g} and is calculated through the box graphs which start at order α_s . The other contribution from which it must be separated is the quark distribution inside the gluon [7]. Mathematically this is expressed as:

$$\sigma^{\gamma^*g}(x, Q^2) = \sigma_h^{\gamma^*g}(x, Q^2/\mu^2) + \Delta q^g(x, \mu^2), \quad (4)$$

where Δq^g is the polarized quark distribution inside a gluon and $\sigma_h^{\gamma^*g}$ is the hard photon-gluon cross section defined, in the IPM, as the contribution coming from quarks in the box graph with transverse momenta greater than the factorization scale.

The full photon-gluon cross section has been calculated to be [9, 10]:

$$\begin{aligned}
\sigma^{\gamma^v g}(x, Q^2) &= -\frac{\alpha_s}{2\pi} N_f \frac{\sqrt{1 - \frac{4m_q^2}{W^2}}}{1 - \frac{4x^2 P^2}{Q^2}} \left[(2x - 1) \left(1 - \frac{2x P^2}{Q^2}\right) \right. \\
&\quad \left. \left(1 - \frac{1}{\sqrt{1 - \frac{4m_q^2}{W^2}} \sqrt{1 - \frac{4x^2 P^2}{Q^2}}} \ln \left(\frac{1 + \sqrt{1 - \frac{4m_q^2}{W^2}} \sqrt{1 - \frac{4x^2 P^2}{Q^2}}}{1 - \sqrt{1 - \frac{4m_q^2}{W^2}} \sqrt{1 - \frac{4x^2 P^2}{Q^2}}} \right) \right) \right. \\
&\quad \left. + \left(x - 1 + \frac{x P^2}{Q^2}\right) \frac{2m_q^2(1 - \frac{4x^2 P^2}{Q^2}) - P^2 x(2x - 1)(1 - \frac{2x P^2}{Q^2})}{m_q^2(1 - \frac{4x^2 P^2}{Q^2}) - P^2 x(x - 1 + \frac{x P^2}{Q^2})} \right] \quad (5)
\end{aligned}$$

with $P^2 = -p^2$ the gluon virtuality, m_q the quark mass and $W^2 = \frac{Q^2(1-x) - P^2 x}{x}$ the invariant mass squared of the photon-gluon system. For very large momentum transfer, $Q^2 \gg m_q^2, P^2$, the full cross section reduces to:

$$\begin{aligned}
\sigma^{\gamma^v g}(x, Q^2/\mu^2) &= \frac{\alpha_s}{2\pi} N_f \left[(2x - 1) \left(\ln \frac{Q^2}{m_q^2 + P^2 x(1 - x)} + \ln \frac{1 - x}{x} - 1 \right) \right. \\
&\quad \left. + (1 - x) \frac{2m_q^2 - P^2 x(2x - 1)}{m_q^2 + P^2 x(1 - x)} \right]. \quad (6)
\end{aligned}$$

It remains to calculate Δq^g . This is given by computing the triangle diagram or, equivalently, the integral over the transverse momentum of the square of the norm of the light-cone $q\bar{q}$ wave function of the gluon [2, 7, 11]. As Δq^g is a soft contribution, the integral over the transverse momentum has to have a cut off:

$$\begin{aligned}
\Delta q^g(x, \mu^2) &= \frac{\alpha_s}{2\pi} N_f \int_0^{\mu^2} dk_{\perp}^2 \frac{m_q^2 + (2x - 1)k_{\perp}^2}{[m_q^2 + P^2 x(1 - x) + k_{\perp}^2]^2} \\
&= \frac{\alpha_s}{2\pi} N_f \left\{ (2x - 1) \ln \left(\frac{\mu^2 + P^2 x(1 - x) + m_q^2}{m_q^2 + P^2 x(1 - x)} \right) \right. \\
&\quad \left. + (1 - x) \frac{2m_q^2 + P^2 x(1 - 2x)}{m_q^2 + P^2 x(1 - x)} \frac{\mu^2}{\mu^2 + P^2 x(1 - x) + m_q^2} \right\}. \quad (7)
\end{aligned}$$

Equation (7) is a generalization of previous results [2, 7] including the dependence on the factorization scale for any values of the quark masses and gluon

virtuality. Its first moment is zero for $\mu^2 \ll m_q^2, P^2$. If $\mu^2 \gg m_q^2, P^2$ the first moment of $\Delta q^g(x, \mu^2)$ is 0 for $P^2 \gg m_q^2$, while it is $\frac{\alpha_s}{2\pi} N_f$ for $m_q^2 \gg P^2$. Using Eqs. (4), (6) and (7) we can calculate the hard gluon coefficient:

$$\begin{aligned} \sigma_h^{\gamma^* g}(x, Q^2, \mu^2) &= \frac{\alpha_s}{2\pi} N_f \left\{ (2x-1) \left[\ln \left(\frac{Q^2}{\mu^2 + P^2 x(1-x) + m_q^2} \right) + \ln \left(\frac{1-x}{x} \right) - 1 \right] \right. \\ &\quad \left. + (1-x) \frac{2m_q^2 + P^2 x(1-2x)}{\mu^2 + m_q^2 + P^2 x(1-x)} \right\}. \end{aligned} \quad (8)$$

Notice that the first moment of Eq. (8) does not depend on the ratio m_q^2/P^2 in the region $\mu^2 \gg m_q^2, P^2$ - it is a legitimate hard contribution. Equation (8) is also a generalization of previous results and from its limit, $\mu^2 \gg m_q^2, P^2$, it may be argued [1] that the gluons contribute to the first moment of $g_1(x, Q^2)$ because $\int_0^1 \sigma_h^{\gamma^* g}(x, Q^2) dx = -\frac{\alpha_s}{2\pi} N_f$.

On the other hand, if one calculates the quark distribution inside a gluon through the triangle graph, which we call Δq_{OPE}^g , using a regularization scheme that respects the axial anomaly, it is found that ¹:

$$\begin{aligned} \Delta q^g(x) - \Delta q_{OPE}^g(x) &= \frac{\alpha_s}{\pi} N_f \left[(2x-1) \ln \left(\frac{\mu^2 + P^2 x(1-x) + m_q^2}{\mu^2} \right) \right. \\ &\quad \left. + \frac{2\mu^2(1-x)}{\mu^2 + P^2 x(1-x) + m_q^2} \right], \end{aligned} \quad (9)$$

where the renormalization scale in the regularization of Δq_{OPE}^g (using \overline{MS}) has been taken to coincide with the factorization scale in the IPM.

Equipped with Eq. (9) we can understand exactly why the hard gluon coefficient in the IPM has a first moment different from zero. The reason is that in the process of factorization the axial anomaly was shifted from the quark distribution inside the gluon to the hard coefficient. Equation (9) reflects the fact that the regularization of Δq_{OPE}^g respects the axial anomaly while the regularization of Δq^g does not. We also see that in the limit $m_q^2 \gg \mu^2$, the discrepancy between the two calculations disappears (at least for the first moment - the x dependence depends on the regularization method). A

¹The triangle graph regularized with a cut off on the transverse momentum results in Eq. (7).

similar phenomenon is found in unpolarized deep inelastic scattering where an analysis by Bass [12] has shown that the trace anomaly induces the same sort of shift when a cut off over the transverse squared momenta of the quarks is used to separate the soft and hard regions.

As a consistency check of our equations, we calculate the OPE hard coefficient, C^g . It is defined in the same way as $\sigma_h^{\gamma^v g}$ in Eq. (4) and calculated with the help of expressions (6) and (9):

$$C^g(x, Q^2/\mu^2) = \frac{\alpha_s}{2\pi} N_f \left[(2x-1) \left(\ln \frac{Q^2}{\mu^2} + \ln \left(\frac{1-x}{x} \right) - 1 \right) + 2(1-x) \right]. \quad (10)$$

This result is independent of mass and its first moment is always zero, in accordance with the results of Kodaira [13] and Bodwin and Qiu [5].

3 Consequences for the First Moment of the Hard Gluon Coefficient

It is interesting to study the dependence on μ^2 of the first moment of $\sigma_h^{\gamma^v g}$. In an early study on this subject², Mankiewicz and Schäfer [22] determined the first moment of the box graph as a function of the minimum transverse momentum carried by the quarks. Their results for $Q^2 \rightarrow \infty$ agree qualitatively with ours, as will soon be seen. But it was also found in Ref. [22] that for momentum transfers of the order of 10 to 100 GeV^2 , the contribution from light quarks³ ($m_q = 10 \text{ MeV}$) is deeply affected by the choice of the minimum value for the transverse quark momentum. In the method used here, such an ambiguity does not exist for the light quarks and its anomalous contribution for $Q^2 = 10$ or 100 GeV^2 is well defined and independent of k_\perp . We use this result to argue that the hard gluon coefficient calculated here is more stable from the point of view of infrared singularities.

Even with the known variations of the anomalous contribution with the factorization scale, it has been widely assumed in the literature [8] that for

²We thank S. Bass for pointing out to us this work.

³We assume for the quark masses their current values. We do not take into account variation of the masses with the factorization scale but note that our conclusions are not significantly altered by small changes in the quark mass.

light quarks (u , d and s) the first moment of $\sigma_h^{\gamma^*g}$ is $-\alpha_s/2\pi$ and for heavy quarks (like c or b) it is zero (because, for $m_q^2 \gg \mu^2$, $\sigma_h^{\gamma^*g}$ reduces to C^g). But it also happens that the gluonic contribution to $g_1(x, Q^2)$ is of the form $\sigma_h^{\gamma^*g}(x, Q^2/\mu^2) \otimes \Delta g(x, \mu^2)$. This means that the scale μ^2 at which the gluon distribution is calculated (or parametrized) is the same scale μ^2 that has to be used in the calculation of the hard gluon coefficient, and that it does not make sense to talk about the magnitude of the hard gluon coefficient without specifying the factorization scale. Thus, the heavy quark contribution is negligible only when the polarized gluon contribution is calculated at a very low scale compared with the quark mass.

In Fig. 1 we show the first moment of $\sigma_h^{\gamma^*g}$ as a function of the factorization scale for the u and d quarks ($m_q^2 \sim 25 \times 10^{-6} \text{ GeV}^2$), for the s quark ($m_q^2 \sim 0.04 \text{ GeV}^2$) and for the c quark ($m_q^2 \sim 9/4 \text{ GeV}^2$). We see that, as is well known [2, 6, 7, 8], the c quark does not contribute when $m_c^2 \gg \mu^2$, as one can also verify directly from Eq. (8). However, for reasonable values of μ^2 there is a contribution large enough to be taken into account. Thus, the significance of the charm contribution to $g_1(x, Q^2)$ depends on where the polarized gluon distribution is calculated. For instance, calculations have been made in the literature using input polarized gluon distributions at a scale of typically 4 GeV^2 . The authors of these calculations usually disregard the charm contribution. We note in passing that in the region of μ^2 where polarized charm can be disregarded, the polarized strange quarks yield only half of the contribution given by u and d quarks. As we see from Fig. 1, the c quark gives around 64% of the contribution of the light quarks for $\mu^2 \sim 4 \text{ GeV}^2$ and so it should not be disregarded if the gluon distribution is calculated at this scale. We also see from Fig. 1 that the u and d quarks give the same contribution, independent of the factorization scale. We further notice that, for practical purposes, the hard gluon coefficient is independent of the exact value of the gluon virtuality P^2 .

The discussion of the preceeding paragraph was based on the not so realistic assumption that the momentum transfer Q^2 is infinitely bigger than any other scales in the theory. It implies, for instance, that when integrating the hard cross section we allow x to go from zero to one. But from simple kinematic arguments we know that x has a maximum value of $x_{max} = Q^2/(Q^2 + P^2 + 4m_q^2)$ and so $x_{max} \rightarrow 1$ only when $Q^2 \gg m_q^2, P^2$. For the finite Q^2 of the current experiments, x_{max} never reaches 1 and so the integral in x has a cut off. For instance if one calculates the first moment

of $\sigma_h^{\gamma^v g}$ for the c quark ($m_q^2 = 9/4 \text{ GeV}^2$) at $\mu^2 = Q^2 = 4 \text{ GeV}^2$, using Eq. (8), one finds that its value changes from -0.64, when x is artificially allowed to reach 1, to ~ 0.015 when the physical cut off in x is applied. What happens is that expression (8) itself was obtained under the assumption of an infinitely large Q^2 . To be more consistent when dealing with finite Q^2 , one should derive the hard cross section from the full cross section without any approximation.

In the general case we then write:

$$C^g = \sigma^{\gamma v g} - \Delta q_{OPE}^g, \quad (11)$$

$$\sigma_h^{\gamma^v g} = \sigma^{\gamma v g} - \Delta q^g, \quad (12)$$

with $\sigma^{\gamma v g}$ given by Eq. (5) and Δq^g and Δq_{OPE}^g given by Eqs. (7) and (9). We stress that these equations are the complete result at order α_s . In Figs. 2 and 3 we show the first moment of $\sigma_h^{\gamma^v g}$, defined in Eq. (12), as a function of the factorization scale μ^2 for $Q^2 = 10 \text{ GeV}^2$ and $Q^2 = 3 \text{ GeV}^2$, respectively. These values were chosen because they are the average Q^2 of the EMC [19, 20] and SLAC [21] experiments. The resulting dependence is very interesting. It shows that in the region of interest ($\mu^2 \geq 1 \text{ GeV}^2$) there is no appreciable dependence on the gluon virtuality or on μ^2 (at least for $Q^2 = 10 \text{ GeV}^2$) but the mass dependence is strong. Remarkably, the contribution from the s quark is never the same as the contribution from the u and d quarks, contrary to what is usually claimed. The s contribution is $\sim 0.9 \alpha_s(Q^2)/2\pi$ for the EMC data and $\sim 0.75 \alpha_s(Q^2)/2\pi$ for the E143-SLAC data. We also find a nonnegligible contribution coming from the c quarks. For the EMC data, the c quark contributes with $\sim 0.2 \alpha_s(Q^2)/2\pi$ and for the E143-SLAC data with $\sim 0.1 \alpha_s(Q^2)/2\pi$. Figure 2 is unaltered ⁴ if we go from $m_q^2 = 0$ to $m_q^2 = 1 \times 10^{-4} \text{ GeV}^2$. If we then compare our Fig. 2 with Fig. 2 of Ref. [22] we see clearly that the present approach does not have a convergence problem in Q^2 and yields a perfectly unambiguous contribution from the light quarks. Finally, we show in Fig. 4 the Q^2 dependence of the polarized charm contribution calculated with $\mu^2 = 3 \text{ GeV}^2$. This contribution, obviously, tends to the value calculated in Fig. 1 ($\sim -0.57 \alpha_s(Q^2)/2\pi$).

⁴In reality, there is a $\sim 1\%$ correction for $\mu^2 \sim 0.01 \text{ GeV}^2$. This result is in complete accord with the fact that the anomalous contribution for the u and d quarks goes to zero as μ^2 goes to zero.

4 Relevance in Analysing the Fraction of Nucleon Spin Carried by Gluons

In terms of the polarized quark and gluon distributions, $g_1^p(x, Q^2)$ for 4 flavors is written as:

$$g_1^p(x, Q^2) = \frac{1}{12}\Delta q_3(x, Q^2) + \frac{1}{36}\Delta q_8(x, Q^2) - \frac{1}{36}\Delta q_{15}(x, Q^2) + \frac{5}{36}\Delta\Sigma(x, Q^2) + \frac{5}{36}\sigma_h^{\gamma^*g}(x, Q^2, \mu^2) \otimes \Delta g(x, \mu^2), \quad (13)$$

where $\Delta q_3 = \Delta u - \Delta d$, $\Delta q_8 = \Delta u + \Delta d - 2\Delta s$, $\Delta q_{15} = \Delta u + \Delta d + \Delta s - 3\Delta c$ and $\Delta\Sigma = \Delta u + \Delta d + \Delta s + \Delta c$. For 3 flavors the coefficient of the singlet part changes from $5/36$ to $1/9$ and Δq_{15} does not exist. To order $\alpha_s(Q^2)$ [13], the first moment of (13) is:

$$\Gamma_1^p(Q^2) = I_3(Q^2) + I_8(Q^2) - I_{15}(Q^2) + I_0(Q^2) - \frac{5}{36} \left(2\frac{\alpha_s(Q^2)}{2\pi} + s_1\frac{\alpha_s(Q^2)}{2\pi} + c_1\frac{\alpha_s(Q^2)}{2\pi} \right) \Delta g(Q^2). \quad (14)$$

The coefficients of $\alpha_s(Q^2)$ have the following meaning. The 2 indicates that the u and d quarks give the same contribution $\frac{\alpha_s(Q^2)}{2\pi}$, as discussed before. The s_1 and c_1 factors give the amount of strange and charm quark contributions, according to Eq. (12) and Figs. 1-3.

To extract the value of $\Delta G(Q^2)$ we will closely follow Refs. [15, 17]. For the sake of comparison, we begin with only 3 flavors and with the common assumption that the u , d and s quarks give the same anomalous contribution.

Under the assumption that the polarized sea originates exclusively from the anomalous gluon contribution we have, for 3 flavors, the following identities:

$$I_3 = \frac{1}{12}(F + D)\left(1 - \frac{\alpha_s}{\pi}\right) \\ I_8 + I_0 = \frac{1}{36}(3F - D) \left[\left(1 - \frac{\alpha_s}{\pi}\right) + 4\left(1 - \frac{\alpha_s}{3\pi}\right) \right], \quad (15)$$

where the quark spin fractions were expressed in terms of the F and D couplings and corrections from the two loop expansion of the beta function and anomalous dimension were incorporated. In NLO , α_s is given as the solution of the following transcendental equation:

$$\ln \frac{Q^2}{\Lambda^2} = \frac{4\pi}{\beta_0 \alpha_s} - \frac{\beta_1}{\beta_0^2} \ln \left[\frac{4\pi}{\beta_0 \alpha_s} + \frac{\beta_1}{\beta_0^2} \right], \quad (16)$$

with $\beta_0 = 11 - 2N_f/3$ and $\beta_1 = 102 - 38N_f/3$. We use $\Lambda = \Lambda^{(3)} = 248 \text{ MeV}$, determined by fixing $\Lambda^{(4)} = 200 \text{ MeV}$ [18]. Using the experimental values of F and D as given in [17], we determine I_3 and $I_8 + I_0$ at $Q^2 = 10 \text{ GeV}^2$ (with $\alpha_s(Q^2 = 10 \text{ GeV}^2) \simeq 0.209$):

$$\begin{aligned} I_3 &= 0.0977 \pm 0.001 \\ I_8 + I_0 &= 0.0779 \pm 0.002 \end{aligned} \quad (17)$$

We now use Eq. (14) to determine $\Delta G(Q^2)$. On the left hand side, we use the experimental result [20]:

$$\Gamma_1^p(Q^2 = 10 \text{ GeV}^2) = 0.142 \pm 0.008 \pm 0.011. \quad (18)$$

On the right hand side we use the results (17), $s_1 = 1$, $c_1 = 0$ and remember that for 3 flavors the singlet coefficient is $1/9$ and $I_{15} = 0$. The result is:

$$\Delta g(Q^2 = 10 \text{ GeV}^2) = 3.04 \pm 1.4. \quad (19)$$

For 4 flavors the analysis is similar. One just has to redefine the integral of $g_1^p(x, Q^2)$:

$$\begin{aligned} I_3 &= \frac{1}{12}(F + D)\left(1 - \frac{\alpha_s}{\pi}\right) \\ I_8 + I_0 - I_{15} &= \frac{5}{36}(3F - D)\left(1 - \frac{\alpha_s}{3\pi}\right). \end{aligned} \quad (20)$$

We then proceed as before and calculate ΔG using the result for the gluon coefficient as displayed in Fig.2. We see that for $\mu^2 = Q^2 = 10 \text{ GeV}^2$, $s_1 \sim 0.9$, $c_1 \simeq 0.21$ and $\alpha_s(Q^2 = 10 \text{ GeV}^2) = 0.2142$, resulting in:

$$\begin{aligned}
I_3 &= 0.0976 \pm 0.001 \\
I_8 + I_0 - I_{15} &= 0.0786 \pm 0.002 \\
\Delta g(Q^2 = 10 \text{ GeV}^2) &= 2.32 \pm 1.06
\end{aligned} \tag{21}$$

In passing we notice that if the usual assumption of infinite momentum transfer were used, then according to the results of Fig.1, at 10 GeV^2 $s_1 = 1$, $c_1 \simeq 0.81$ and hence $\Delta g \simeq 1.89$.

5 The x dependence

The exact x dependence of the anomalous contribution is a matter of convention because the freedom in the factorization scheme while calculating $\sigma_h^{\gamma^*g}$. Other choices of regularization would result in different functions of x . But, as shown by Glück et al. [14], the exact form of the x dependence seems not to be very important. Once we do not know the form of the polarized gluon distribution, the best we can do is constrain it by some general considerations. For instance, there is the positivity condition:

$$|\Delta g(x, Q^2)| \leq g(x, Q^2), \tag{22}$$

where $g(x, Q^2)$ is the unpolarized gluon distribution. A very simple form that satisfies the above condition is:

$$\Delta(x) = x^\alpha g(x), \tag{23}$$

where α is determined through the normalization of Δg . For Δg of Eq. (21), $\alpha = 0.49$. The advantage of using this form to study the x dependence is its simplicity. The problem with Eq. (23) is that it does not have the correct behavior as $x \rightarrow 0$. As proposed by Brodsky et al. [23],

$$\frac{\Delta g(x)}{g(x)} \rightarrow x, \tag{24}$$

as $x \rightarrow 0$. From the many ways to satisfy both conditions (22) and (24), we choose:

$$\Delta g(x, \mu^2 = 9 \text{ GeV}^2) = \alpha x g(x, \mu^2 = 9 \text{ GeV}^2) (1 - x)^3, \tag{25}$$

where $\alpha = 6.92$ for $\Delta g = 2.32$. We made this choice guided only by the desire of simplicity and to produce a polarized gluon distribution that resembles an already existing one [15]. For the unpolarized gluon distribution, we use the one given by the NMC [24], determined from inelastic J/ψ production:

$$xg(x) = \frac{1}{2}(\eta + 1)(1 - x)^\eta. \quad (26)$$

This parametrization is valid for $\mu^2 = 9 \text{ GeV}^2$ and should not be trusted for $x \leq 0.01$. Again, this choice is based on simplicity and we note that a further parametrization by the NMC [25] group agrees with Eq. (26) for $x \geq 0.01$. The parameter η is $\eta = 5.1 \pm 0.9$. Given these choices, we show in Fig. 5 the forms (23) and (25) for the polarized gluon distributions plus the forms of Brodsky et al. [23] and Gehrmann and Stirling (GS) [15], calculated at 4 GeV^2 . Our parametrization (25) is slightly higher than that of GS because of the normalization factor. If we use the same normalization as theirs⁵, both curves would be essentially the same. Evolution from 4 to 9 GeV^2 for the GS distribution also has small effects. The parametrization of Brodsky et al. [23] is much smaller than the others because in their approach the polarized gluons are not responsible for the small experimental value of Eq. (18).

Using the constructed gluon distributions, we can estimate where in x the anomalous contribution is located. In Fig. 6 we show the anomalous contribution, $\frac{5}{36}\sigma_h^{\gamma^*g}(x, Q^2, \mu^2) \otimes \Delta g(x, \mu^2)$, for $\Delta g(x) = \alpha xg(x)(1 - x)^3$. We see that its contribution inside the experimental region is important. To better evaluate its importance, we calculated the amount of the total gluon that lies inside the region $0.01 \leq x \leq 1$. For 4 flavors, the contribution from $x \geq 0.01$ corresponds to about 66% of the total anomalous contribution. In the case of 3 flavors, this percentage is $\sim 69\%$. For $\Delta g(x) = x^\alpha g(x)$ this conclusion is not dramatically altered.

It is also interesting to compare the anomalous gluon contribution directly with the experimental data. To this end, we plot in Fig. 7 the experimental data [19, 20] for $g_1^p(x)$ together with an early next-to-leading order estimate [26] for the valence quark distribution and the anomalous gluon contribution for the case of 3 and 4 flavors. A remark is necessary here. The calculation of the hard gluon coefficient was performed through a cut off on the transverse

⁵We note that in [15], the coupling constant is calculated in leading order rather than in NLO. This would lead to an increase of the total polarization carried by the gluons.

momenta of the partons in order to regularize the integrals. This procedure is the definition of the parton model. On the other hand, we calculated the strong coupling constant, and also the evolution of g_1^p in Ref. [26], using the \overline{MS} scheme. In principle, showing the x dependence of two quantities in different schemes is not a consistent procedure. However, the problem is not as bad as it looks. First, if we change schemes we can maintain α_s unaltered by a simple redefinition of the parameter Λ in expression (16). Second, the theoretical curve for $g_1^p(x, Q^2)$ is to be interpreted as a guide of what a parametrization for the valence part of the polarized structure function would give, the regions in which it differs from the data and where it should be corrected. A proper procedure would be to calculate the quark distribution, the anomalous dimensions and the Wilson coefficients in the same scheme. That said we proceed noticing that the integral over x of the valence contribution calculated in [26] (~ 0.169) is in complete agreement with the estimates calculated previously in Section 4. The two curves below the origin, are the anomalous contributions that should be added to the solid curve for $N_f=3$ or 4. As we fixed the normalization of the total polarized glue for either 3 or 4 flavors, there is no noticeable difference between the two cases. We see that the anomalous contribution is potentially important to correct the x dependence of the polarized valence distribution inside the proton.

6 Discussion

In summary, there is a gluonic contribution to the proton spin when the IPM hard gluon coefficient is defined through Eq. (12) with Δq^g defined as the quark distribution inside the gluon with transverse squared momentum less than the factorization scale. As a consequence, this anomalous gluonic contribution in the IPM is free of infrared ambiguities. We showed that if we accept the commonly used assumption of an infinitely big momentum transfer, there is a c quark contribution to the spin in addition to the u , d and s quark contributions. The contribution from the massive quarks is dependent on the factorization scale at which the polarized gluon distribution is calculated. The c quark contribution is small only in the region $\mu^2 < 1 \text{ GeV}^2$, in which case the s quark contribution is also strongly affected.

We also calculated what would be the possible anomalous corrections

when the momentum transfer is in the region of the present experiments. To perform such a calculation we have to keep all terms in m_q^2/Q^2 and P^2/Q^2 in the full photon-gluon cross section when calculating the hard gluon coefficient. This means that we are including higher twist effects and, although we use the complete result at order $\alpha_s(Q^2)$, possible corrections coming from higher order terms in $\alpha_s(Q^2)$ could be important and so our calculation is incomplete. Even so, we think that our results are more consistent than simply using the approximate expression (8) for the hard gluon coefficient in the case of massive quarks and relatively low Q^2 . The corrections due to finite Q^2 are not small and we think they should be taken into account when calculating the amount of spin carried by gluons. When studying the x dependence of the anomalous contribution, we conclude that both 3 and 4 flavors give approximately the same contribution inside the experimental region. But the amount of polarized glue needed to fit the data is much smaller when charm is included. Moreover, we showed that from the conceptual point of view, it would be wrong not to include a fourth flavor.

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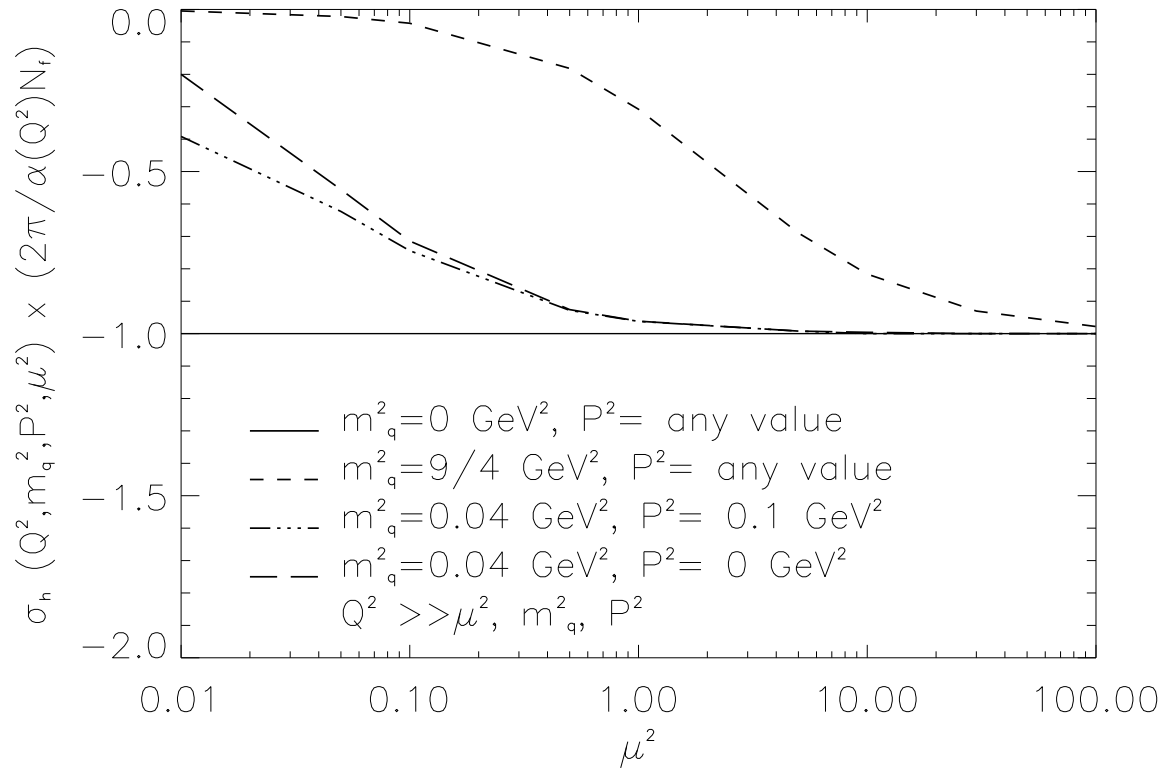


Figure 1: Hard gluon coefficient as given by Eq. (8), calculated with the assumption of infinite momentum transfer as a function of the factorization scale. For realistic scales ($\mu^2 > 1 \text{ GeV}^2$), the charm contribution is seen to be important.

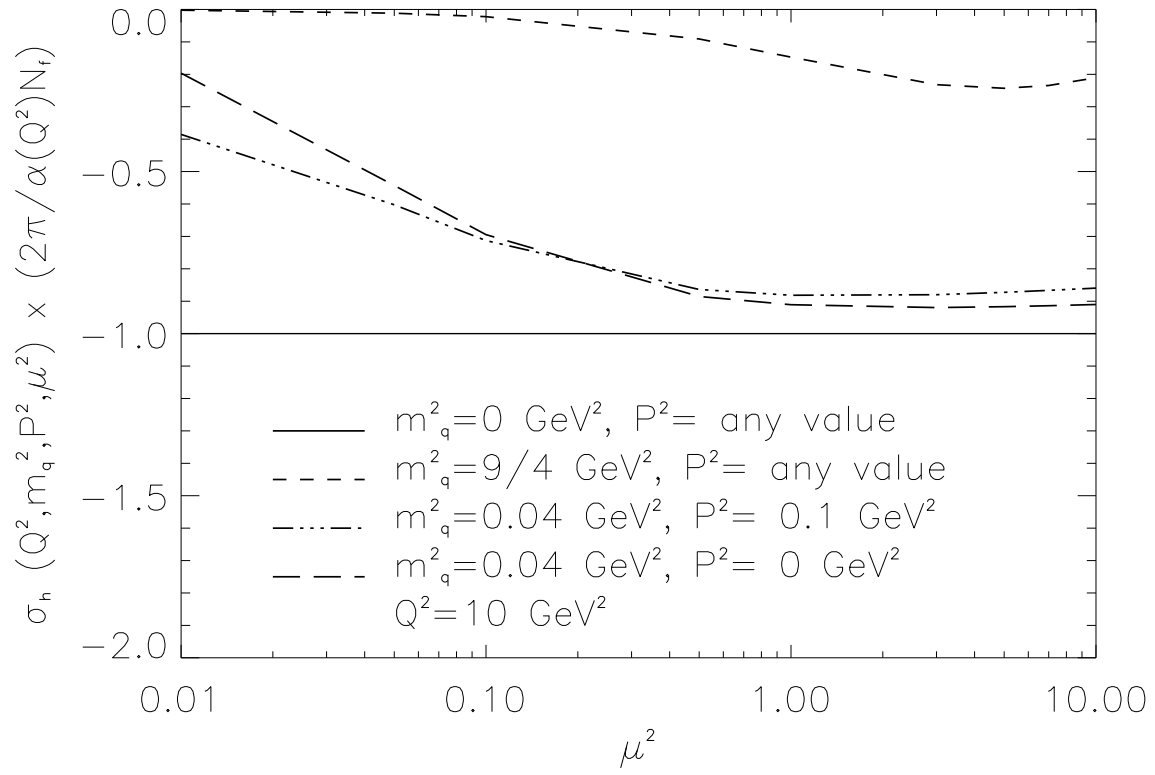


Figure 2: Hard gluon coefficient as given by Eqs. (5), (7) and (12). The momentum transfer is fixed at 10 GeV^2 . It is seen that the strange quark contribution never equals that from the up and down quarks and the charm quark contribution is sizable for $\mu^2 > 1 \text{ GeV}^2$.

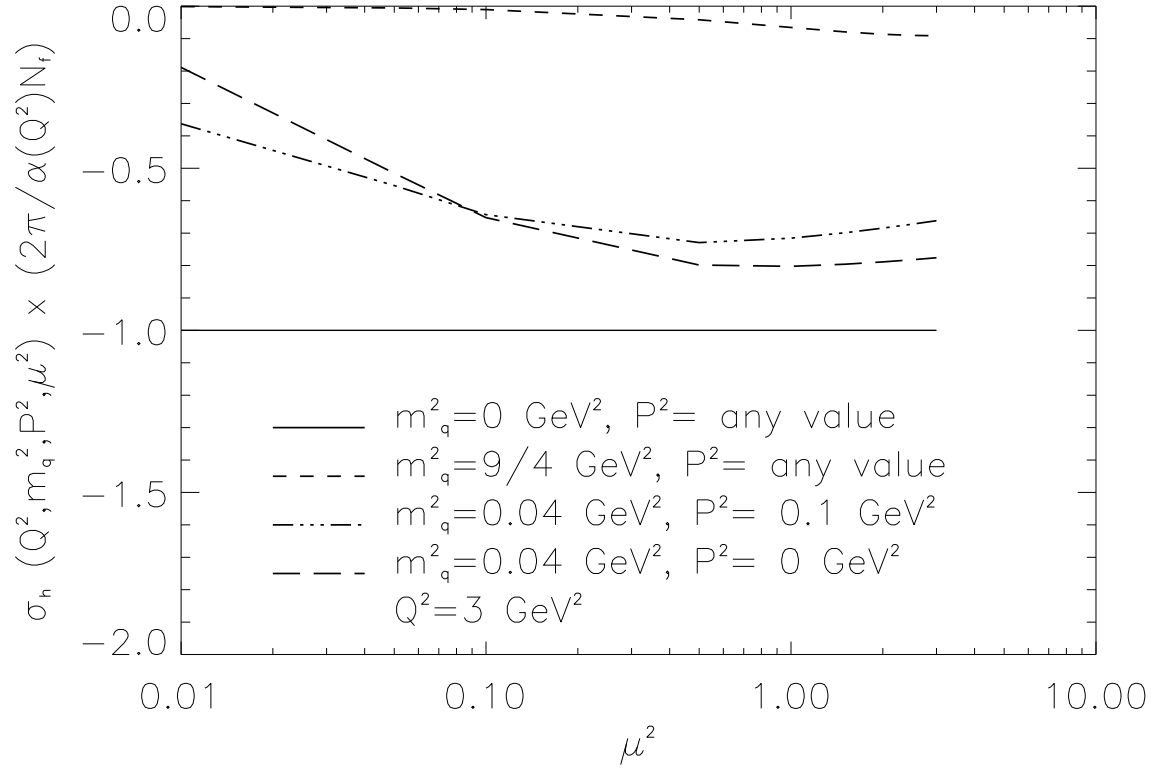


Figure 3: Hard gluon coefficient as given by Eqs. (5), (7) and (12). The momentum transfer is fixed at 3 GeV^2 . It is seen that the strange quark contributes with approximately 75% of the up and down quarks in the realistic region of $\mu^2 > 1 \text{ GeV}^2$. In the same region, the charm quark gives a 10% contribution.

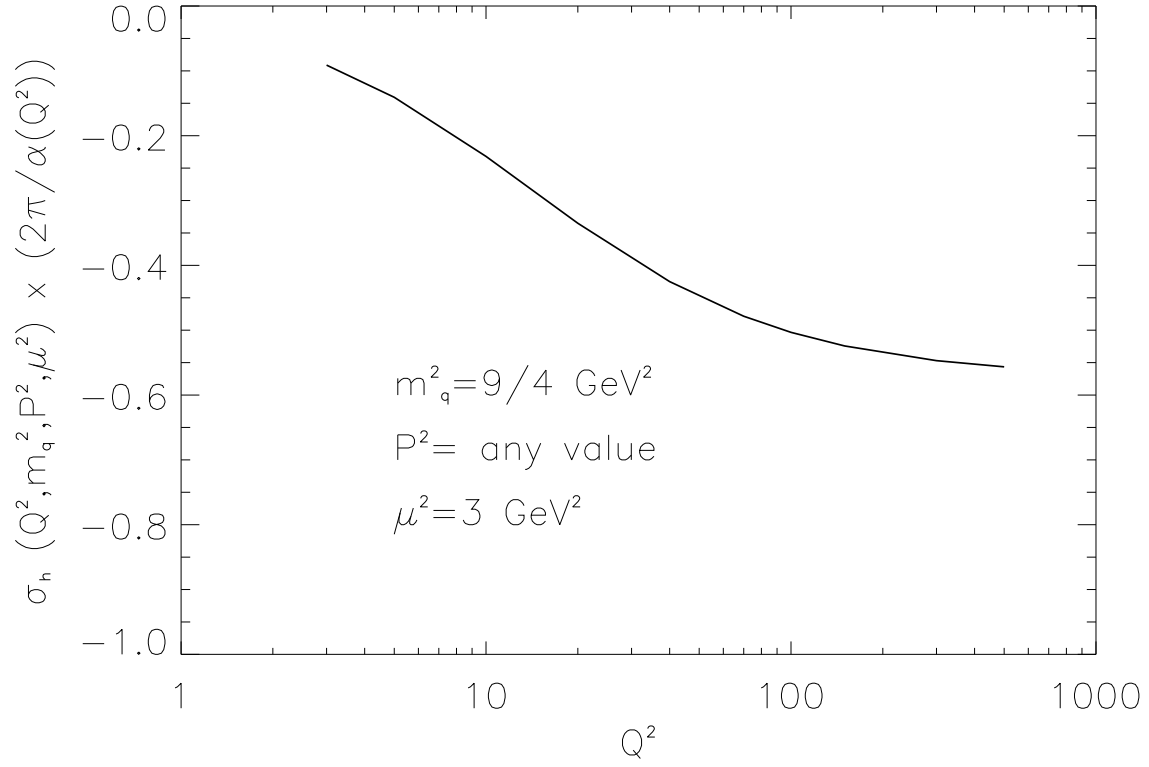


Figure 4: Hard gluon coefficient for the charm quark, calculated with Eqs. (5), (7) and (12). The factorization scale is fixed at 3 GeV^2 and the Q^2 dependence is studied.

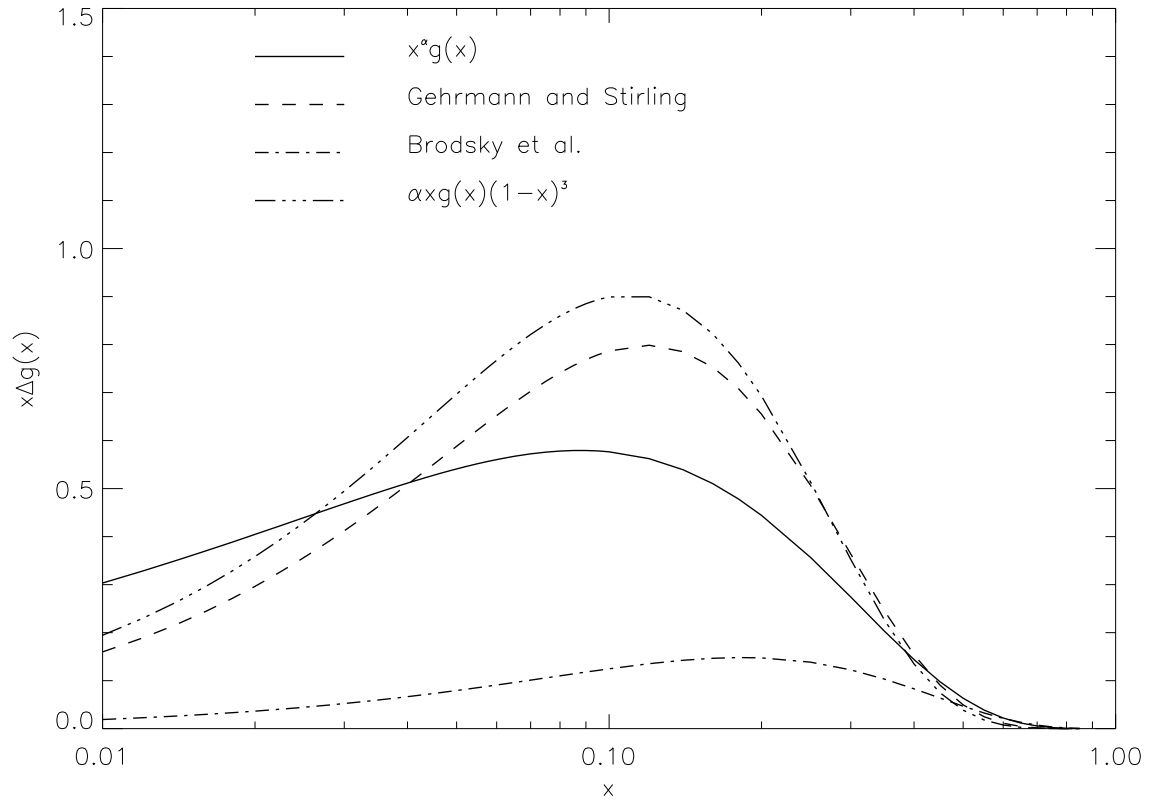


Figure 5: Comparison of various polarized gluon distributions considered in the text.

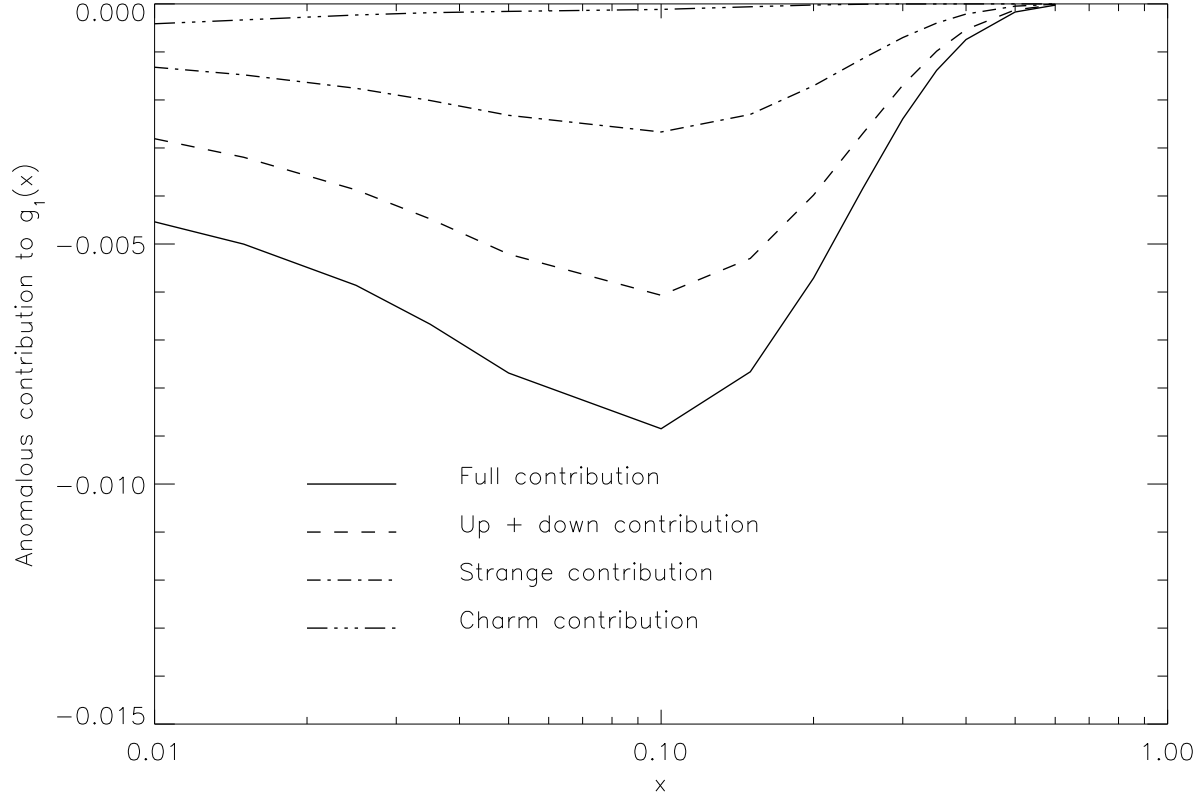


Figure 6: Comparison of the x -dependence of the non-strange, strange and charm quark distributions to $g_1(x)$. The anomalous contribution is given by $\frac{5}{36}\sigma_h^{\gamma^*g}(x, Q^2, \mu^2) \otimes \Delta g(x, \mu^2)$ and it is used the form $\Delta g(x, \mu^2 = 9 \text{ GeV}^2) = \alpha x g(x)(1-x)^3$ for the polarized gluon.

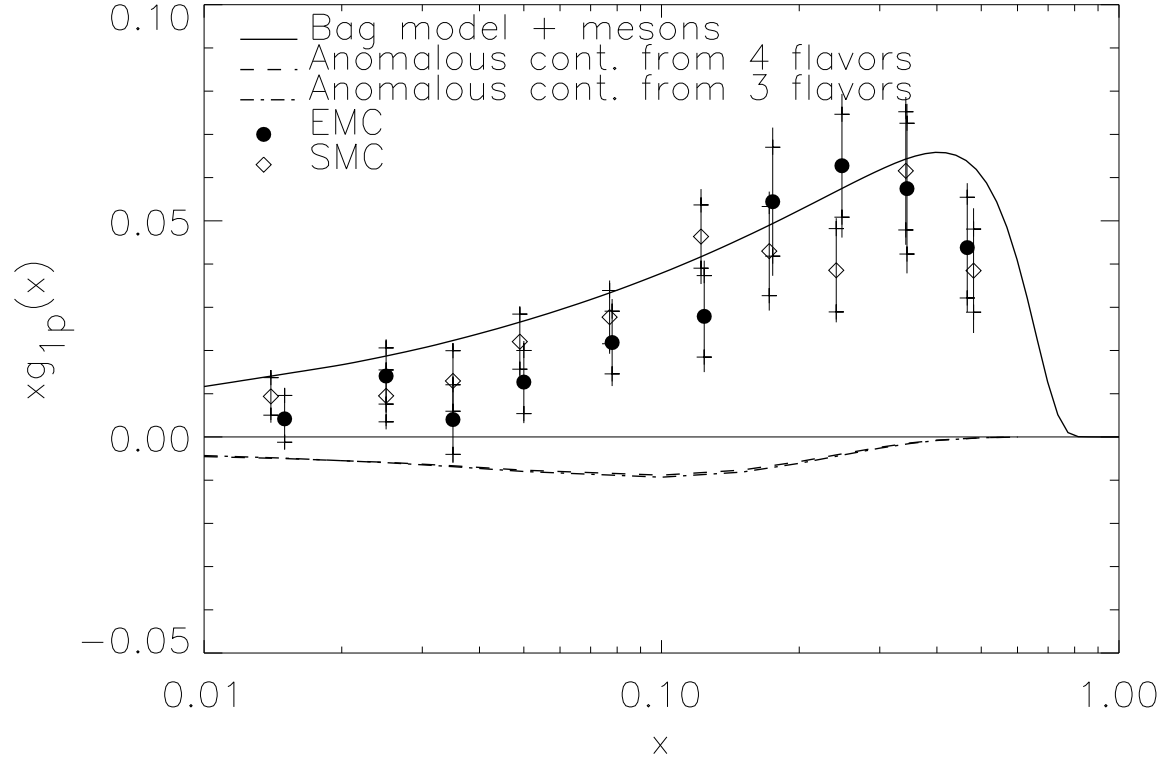


Figure 7: EMC [19] and SMC [20] data for $g_{1p}(x)$ at 10 GeV^2 . The theoretical curve for the polarized valence distribution is calculated in NLO and taken from Ref.[26]. The anomalous contribution should be subtracted is to be subtracted from the theoretical curve.